Combining approximate zero constraints for measurement invariance and cross-loadings: An application of dual process growth curve models with panel data

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Meeting of the Working Group Structural Equation Modeling
Zurich, 2016
1. Bayesian SEM
   - Measurement invariance
   - Cross-loadings

2. Illustration
   - Substantive question
   - Data
   - Results I: cross-loadings
   - Results II: measurement invariance
   - Results III: final model
Bayesian SEM

- Measurement invariance
- Cross-loadings

Illustration

- Substantive question
- Data
- Results I: cross-loadings
- Results II: measurement invariance
- Results III: final model
\[ Y_{k,t} = T_{k,t} + \Lambda_{k,t}\eta_t + \Theta_{\varepsilon_k,t} \quad k = 1, \ldots, K; t = 1, \ldots, T \]
\[ Y_{k,t} = T_{k,t} + \Lambda_{k,t}\eta_t + \Theta\epsilon_{k,t} \quad k = 1, \ldots, K; t = 1, \ldots, T \quad (1) \]

(a) Exact MI in \( \Lambda_{k,t} \):

\[
\begin{align*}
\lambda_{1,1} &= \lambda_{1,2} = \ldots = \lambda_{1,T} \\
\lambda_{2,1} &= \lambda_{2,2} = \ldots = \lambda_{2,T} \\
\vdots &= \vdots = \ldots = \vdots \\
\lambda_{K,1} &= \lambda_{K,2} = \ldots = \lambda_{K,T}
\end{align*}
\]

Highest level of stringency

Differences across group/time exactly zero
Bayesian SEM
Measurement invariance

\[ Y_{k,t} = T_{k,t} + \Lambda_{k,t}\eta_t + \Theta\varepsilon_{k,t} \quad k = 1, \ldots, K; t = 1, \ldots, T \quad (1) \]

(a) Exact MI in \( \Lambda_{k,t} \):

\[ \lambda_{1,1} = \lambda_{1,2} = \ldots = \lambda_{1,T} \]
\[ \lambda_{2,1} = \lambda_{2,2} = \ldots = \lambda_{2,T} \]
\[ \vdots = \vdots = \ldots = \vdots \]
\[ \lambda_{K,1} = \lambda_{K,2} = \ldots = \lambda_{K,T} \]

- Highest level of stringency
- Differences across group/time exactly zero

(b) Approximate MI in \( \Lambda_{k,t} \):

\[ \lambda_{1,1} \approx \lambda_{1,2} \approx \ldots \approx \lambda_{1,T} \]
\[ \lambda_{2,1} \approx \lambda_{2,2} \approx \ldots \approx \lambda_{2,T} \]
\[ \vdots \approx \vdots \approx \ldots \approx \vdots \]
\[ \lambda_{K,1} \approx \lambda_{K,2} \approx \ldots \approx \lambda_{K,T} \]

- Flexibility, “wiggle room” (Van de Schoot et al., 2013)
- Identification of non-invariants: “two-step Bayesian analysis procedure” (Muthén & Asparouhov, 2013)
Bayesian SEM
Prior distributions for approximate zero constraints

Figure 1: Priors for exact (a) and approximate (b) MI

Prior for exact zero differences in $\lambda$s between groups/time points

$\lambda_{\text{diff}} \sim N(0,0.00)$

Prior for approximate zero differences in $\lambda$s between groups/time points

$\lambda_{\text{diff}} \sim N(0,0.01)$

95%
1 Bayesian SEM
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Bayesian SEM

Cross-loadings

- **Exact zero cross-loadings**

\[
\begin{pmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
  y_4 \\
  y_5 \\
  y_6 \\
  y_7 \\
  y_8
\end{pmatrix} =
\begin{pmatrix}
  \lambda_{y_{11}} = 1 & \lambda_{y_{12}} = 0 \\
  \lambda_{y_{21}} & \lambda_{y_{22}} = 0 \\
  \lambda_{y_{31}} & \lambda_{y_{32}} = 0 \\
  \lambda_{y_{41}} & \lambda_{y_{42}} = 0 \\
  \lambda_{y_{51}} = 0 & \lambda_{y_{52}} = 1 \\
  \lambda_{y_{61}} = 0 & \lambda_{y_{62}} \\
  \lambda_{y_{71}} = 0 & \lambda_{y_{72}} \\
  \lambda_{y_{81}} = 0 & \lambda_{y_{82}}
\end{pmatrix}
\ast
\begin{pmatrix}
  \eta_1 \\
  \eta_2
\end{pmatrix}
+ \begin{pmatrix}
  \varepsilon_1 \\
  \varepsilon_2 \\
  \varepsilon_3 \\
  \varepsilon_4 \\
  \varepsilon_5 \\
  \varepsilon_6 \\
  \varepsilon_7 \\
  \varepsilon_8
\end{pmatrix}
\]

(4)
Bayesian SEM

Cross-loadings

- **Exact zero cross-loadings**

\[
\begin{pmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
  y_4 \\
  y_5 \\
  y_6 \\
  y_7 \\
  y_8
\end{pmatrix} = \begin{pmatrix}
  \lambda_{y11} = 1 & \lambda_{y12} = 0 \\
  \lambda_{y21} & \lambda_{y22} = 0 \\
  \lambda_{y31} & \lambda_{y32} = 0 \\
  \lambda_{y41} & \lambda_{y42} = 0 \\
  \lambda_{y51} = 0 & \lambda_{y52} = 1 \\
  \lambda_{y61} = 0 & \lambda_{y62} \\
  \lambda_{y71} = 0 & \lambda_{y72} \\
  \lambda_{y81} = 0 & \lambda_{y82}
\end{pmatrix} \ast \begin{pmatrix}
  \eta_1 \\
  \eta_2
\end{pmatrix} + \begin{pmatrix}
  \varepsilon_1 \\
  \varepsilon_2 \\
  \varepsilon_3 \\
  \varepsilon_4 \\
  \varepsilon_5 \\
  \varepsilon_6 \\
  \varepsilon_7 \\
  \varepsilon_8
\end{pmatrix}
\] (4)

- **Approximate zero cross-loadings**

\[
\begin{pmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
  y_4 \\
  y_5 \\
  y_6 \\
  y_7 \\
  y_8
\end{pmatrix} = \begin{pmatrix}
  \lambda_{y11} = 1 & \lambda_{y12} \approx 0 \\
  \lambda_{y21} & \lambda_{y22} \approx 0 \\
  \lambda_{y31} & \lambda_{y32} \approx 0 \\
  \lambda_{y41} & \lambda_{y42} \approx 0 \\
  \lambda_{y51} \approx 0 & \lambda_{y52} = 1 \\
  \lambda_{y61} \approx 0 & \lambda_{y62} \\
  \lambda_{y71} \approx 0 & \lambda_{y72} \\
  \lambda_{y81} \approx 0 & \lambda_{y82}
\end{pmatrix} \ast \begin{pmatrix}
  \eta_1 \\
  \eta_2
\end{pmatrix} + \begin{pmatrix}
  \varepsilon_1 \\
  \varepsilon_2 \\
  \varepsilon_3 \\
  \varepsilon_4 \\
  \varepsilon_5 \\
  \varepsilon_6 \\
  \varepsilon_7 \\
  \varepsilon_8
\end{pmatrix}
\] (5)

Seddig, Daniel (UZH)  Approximate MI and zero CLs  07.04.2016, Zurich
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- Results III: final model
Illustration

Hedonism and associating with delinquent peer groups in adolescence
Hedonism and associating with delinquent peer groups in adolescence

- **Hedonism**: Pleasure and sensuous gratification
- **Stimulation**: Excitement, novelty, and challenge in life
- **Hedonism/Stimulation ↔ Delinquent Peer Groups**
- **Development**: as adolescents interest in both dimensions decreases, associations with delinquent peer groups decreases
Bayesian SEM

- Measurement invariance
- Cross-loadings

Illustration

- Substantive question
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  - Results III: final model
“Crimoc-study”; German criminological panel study

Panel data; $n=357$ male respondents; ages 14 to 20

Beliefs about hedonism/stimulation (scaled 1-5):
- $h_1$: understanding for people who do what they desire
- $h_2$: need for excitement
- $h_3$: living a life of pleasure

Association with violent peer group (scaled 1-5):
- $g_1$: group enforces interests with force
- $g_2$: group involved in brawls
1 Bayesian SEM
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   • Cross-loadings

2 Illustration
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   • Data
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Illustration
Cross-loadings: BCFA

Figure 2: CFA with cross-loadings
Table 1: BCFA model assessment for age 18 (n=357)

<table>
<thead>
<tr>
<th>Prior ($\lambda_{CL}$)</th>
<th>BIC</th>
<th>DIC</th>
<th>PPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim N(0,0.000)$</td>
<td>5021</td>
<td>4958</td>
<td>0.030</td>
</tr>
<tr>
<td>$\sim N(0,0.001)$</td>
<td>5045</td>
<td>4953</td>
<td>0.102</td>
</tr>
<tr>
<td>$\sim N(0,0.010)$</td>
<td>5033</td>
<td>4944</td>
<td>0.453</td>
</tr>
<tr>
<td>$\sim N(0,0.050)$</td>
<td>5031</td>
<td>4943</td>
<td>0.509</td>
</tr>
<tr>
<td>$\sim N(0,0.100)$</td>
<td>5031</td>
<td>4943</td>
<td>0.512</td>
</tr>
</tbody>
</table>

Note: BIC = Bayesian information criterion; DIC = deviance information criterion; PPP = posterior predictive p-value.
### Illustration

Cross-loadings: BCFA with $\sim \mathcal{N}(0,0.010)$

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Posterior S.D.</th>
<th>One-Tailed P-Value</th>
<th>95% C.I. Lower 2.5%</th>
<th>95% C.I. Upper 2.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>group_18 BY</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g1_18</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>g2_18</td>
<td>0.856</td>
<td>0.159</td>
<td>0.000</td>
<td>0.652</td>
<td>1.242</td>
</tr>
<tr>
<td>h1_18</td>
<td>-0.063</td>
<td>0.087</td>
<td>0.233</td>
<td>-0.235</td>
<td>0.104</td>
</tr>
<tr>
<td>h2_18</td>
<td>0.152</td>
<td>0.079</td>
<td>0.034</td>
<td>-0.012</td>
<td>0.297</td>
</tr>
<tr>
<td>h3_18</td>
<td>-0.012</td>
<td>0.076</td>
<td>0.438</td>
<td>-0.167</td>
<td>0.133</td>
</tr>
<tr>
<td><strong>h1_18 BY</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h1_18</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>h2_18</td>
<td>0.563</td>
<td>0.168</td>
<td>0.000</td>
<td>0.293</td>
<td>0.951</td>
</tr>
<tr>
<td>h3_18</td>
<td>0.572</td>
<td>0.153</td>
<td>0.000</td>
<td>0.316</td>
<td>0.919</td>
</tr>
<tr>
<td>g1_18</td>
<td>0.002</td>
<td>0.083</td>
<td>0.492</td>
<td>-0.158</td>
<td>0.168</td>
</tr>
<tr>
<td>g2_18</td>
<td>0.004</td>
<td>0.077</td>
<td>0.477</td>
<td>-0.156</td>
<td>0.151</td>
</tr>
</tbody>
</table>

**STDYX Standardization**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>h1_18 WITH</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>group_18</td>
<td>0.404</td>
<td>0.118</td>
<td>0.002</td>
<td>0.145</td>
<td>0.605</td>
</tr>
</tbody>
</table>
Illustration
Cross-loadings: Posterior distribution of cross-loading for item “h2_18”
1 Bayesian SEM
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Figure 3: LGMs for *hedo* and *group*
Table 2: Univariate LGM assessment with scalar MI (n=357)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Prior ($\lambda_{\text{diff}}$)</th>
<th>BIC</th>
<th>DIC</th>
<th>PPP</th>
<th>BIC</th>
<th>DIC</th>
<th>PPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>$\sim$N(0, 0.000)</td>
<td>12262</td>
<td>12092</td>
<td>0.010</td>
<td>6057</td>
<td>5934</td>
<td>0.381</td>
</tr>
<tr>
<td>Appr.</td>
<td>$\sim$N(0, 0.001)</td>
<td>12337</td>
<td>12073</td>
<td>0.183</td>
<td>6121</td>
<td>5933</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td>$\sim$N(0, 0.010)</td>
<td>12331</td>
<td>12072</td>
<td>0.252</td>
<td>6117</td>
<td>5933</td>
<td>0.543</td>
</tr>
<tr>
<td></td>
<td>$\sim$N(0, 0.050)</td>
<td>12329</td>
<td>12070</td>
<td>0.263</td>
<td>6116</td>
<td>5930</td>
<td>0.545</td>
</tr>
<tr>
<td>Partial</td>
<td>$\sim$N(0, 0.000)</td>
<td>12251</td>
<td>12078</td>
<td>0.071</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: BIC = Bayesian information criterion; DIC = deviance information criterion; PPP = posterior predictive p-value.
<table>
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<tr>
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<th>95% C.I. Lower 2.5%</th>
<th>95% C.I. Upper 2.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>h1_14</strong> by <strong>hedo_14</strong></td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td><strong>h2_14</strong></td>
<td>0.694</td>
<td>0.089</td>
<td>0.531</td>
<td>0.882</td>
</tr>
<tr>
<td><strong>h3_14</strong></td>
<td>0.759</td>
<td>0.099</td>
<td>0.578</td>
<td>0.969</td>
</tr>
<tr>
<td><strong>h1_16</strong> by <strong>hedo_16</strong></td>
<td>0.970</td>
<td>0.042</td>
<td>0.890</td>
<td>1.054</td>
</tr>
<tr>
<td><strong>h2_16</strong></td>
<td>0.745</td>
<td>0.091</td>
<td>0.578</td>
<td>0.935</td>
</tr>
<tr>
<td><strong>h3_16</strong></td>
<td>0.789</td>
<td>0.100</td>
<td>0.608</td>
<td>1.001</td>
</tr>
<tr>
<td><strong>h1_18</strong> by <strong>hedo_18</strong></td>
<td>0.953</td>
<td>0.052</td>
<td>0.855</td>
<td>1.057</td>
</tr>
<tr>
<td><strong>h2_18</strong></td>
<td>0.709</td>
<td>0.096</td>
<td>0.535</td>
<td>0.912</td>
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<tr>
<td><strong>h3_18</strong></td>
<td>0.788</td>
<td>0.106</td>
<td>0.596</td>
<td>1.012</td>
</tr>
<tr>
<td><strong>h1_20</strong> by <strong>hedo_20</strong></td>
<td>0.922</td>
<td>0.065</td>
<td>0.801</td>
<td>1.056</td>
</tr>
<tr>
<td><strong>h2_20</strong></td>
<td>0.702</td>
<td>0.101</td>
<td>0.517</td>
<td>0.917</td>
</tr>
<tr>
<td><strong>h3_20</strong></td>
<td>0.829</td>
<td>0.112</td>
<td>0.625</td>
<td>1.066</td>
</tr>
</tbody>
</table>
### Hedonism: univariate LGM factor loadings with \( \sim \mathcal{N}(0, 0.050) \)

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<th>Upper 2.5%</th>
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<tbody>
<tr>
<td><strong>hedo_14</strong> BY</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h1_14</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>h2_14</td>
<td>0.706</td>
<td>0.097</td>
<td>0.000</td>
<td>0.530</td>
<td>0.910</td>
</tr>
<tr>
<td>h3_14</td>
<td>0.773</td>
<td>0.107</td>
<td>0.000</td>
<td>0.581</td>
<td>0.999</td>
</tr>
<tr>
<td><strong>hedo_16</strong> BY</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>h1_16</td>
<td>0.939</td>
<td>0.082</td>
<td>0.000</td>
<td>0.777</td>
<td>1.101</td>
</tr>
<tr>
<td>h2_16</td>
<td>0.746</td>
<td>0.102</td>
<td>0.000</td>
<td>0.560</td>
<td>0.960</td>
</tr>
<tr>
<td>h3_16</td>
<td>0.780</td>
<td>0.110</td>
<td>0.000</td>
<td>0.581</td>
<td>1.014</td>
</tr>
<tr>
<td><strong>hedo_18</strong> BY</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h1_18</td>
<td>0.962</td>
<td>0.110</td>
<td>0.000</td>
<td>0.742</td>
<td>1.173</td>
</tr>
<tr>
<td>h2_18</td>
<td>0.680</td>
<td>0.119</td>
<td>0.000</td>
<td>0.465</td>
<td>0.929</td>
</tr>
<tr>
<td>h3_18</td>
<td>0.770</td>
<td>0.133</td>
<td>0.000</td>
<td>0.529</td>
<td>1.049</td>
</tr>
<tr>
<td><strong>hedo_20</strong> BY</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h1_20</td>
<td>0.868</td>
<td>0.138</td>
<td>0.000</td>
<td>0.602</td>
<td>1.138</td>
</tr>
<tr>
<td>h2_20</td>
<td>0.663</td>
<td>0.137</td>
<td>0.000</td>
<td>0.416</td>
<td>0.954</td>
</tr>
<tr>
<td>h3_20</td>
<td>0.798</td>
<td>0.156</td>
<td>0.000</td>
<td>0.511</td>
<td>1.119</td>
</tr>
</tbody>
</table>
1 Bayesian SEM
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Figure 4: Multivariate LGM with cross-loadings
Table 3: Multivariate LGM assessment with scalar MI ($n=357$)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Prior ($\lambda_{\text{diff}} / \tau_{\text{diff}}$)</th>
<th>Prior (CLs)</th>
<th>BIC</th>
<th>DIC</th>
<th>PPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact MI w/o CLs</td>
<td>$\sim N(0,0.000)$</td>
<td>$\sim N(0,0.000)$</td>
<td>18232</td>
<td>17971</td>
<td>0.000</td>
</tr>
<tr>
<td>Exact MI w/ CLs</td>
<td>$\sim N(0,0.010)$</td>
<td>$\sim N(0,0.010)$</td>
<td>18267</td>
<td>17918</td>
<td>0.171</td>
</tr>
<tr>
<td>Appr. MI w/o CLs</td>
<td>$\sim N(0,0.010)$</td>
<td>$\sim N(0,0.000)$</td>
<td>18414</td>
<td>17954</td>
<td>0.025</td>
</tr>
<tr>
<td>Appr. MI w/ CLs</td>
<td>$\sim N(0,0.010)$</td>
<td>$\sim N(0,0.010)$</td>
<td>18479</td>
<td>17919</td>
<td>0.372</td>
</tr>
</tbody>
</table>

Note: BIC = Bayesian information criterion; DIC = deviance information criterion; PPP = posterior predictive p-value.
### Multivariate LGM: Estimates

<table>
<thead>
<tr>
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<th>Estimate</th>
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<th>P-Value</th>
<th>Lower 2.5%</th>
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<td></td>
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<td></td>
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</tr>
</tbody>
</table>

**STDYX Standardization**

- **I_HE WITH I_GR**: 0.580
- **S_HE WITH S_GR**: 0.463
### Illustration

#### Multivariate LGM: Estimates

**Means**

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<tr>
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</tbody>
</table>

**STDYX Standardization**

- **I_HE WITH I_GR**
  - Estimate: 0.580
  - S.D.: 0.142
  - P-Value: 0.000
  - Lower 2.5%: 0.284
  - Upper 2.5%: 0.833
  - (ML = 0.778)

- **S_HE WITH S_GR**
  - Estimate: 0.463
  - S.D.: 0.194
  - P-Value: 0.010
  - Lower 2.5%: 0.071
  - Upper 2.5%: 0.828
  - (ML = 0.680)
BSEM useful
but...
- Prior choice may be an obstacle
- Compromise between fit and precision
- Giving up parsimony vs. using prior assumptions
Appendix
Multivariate LGM: Posterior distribution of intercept mean (hedonism)
Appendix
Multivariate LGM: Posterior distribution of slope mean (hedonism)

[Graph showing the posterior distribution of a slope mean with various statistics indicated, including mean, median, mode, and confidence intervals.]

Seddig, Daniel (UZH)
Approximate MI and zero CLs
07.04.2016, Zurich
Appendix
Multivariate LGM: Posterior distribution of intercept mean (peer group)
Appendix
Multivariate LGM: Posterior distribution of slope mean (group)
Appendix
Multivariate LGM: Posterior distribution of intercept correlation

Seddig, Daniel (UZH)  Approximate MI and zero CLs  07.04.2016, Zurich
Appendix

Multivariate LGM: Posterior distribution of slope correlation
Analysis:
Estimator = Bayes;
Chains = 2;
Proc = 2;
Biterations = 1000000(200000);
Bseed = 3010;
Figure 5: Potential scale reduction factor (PSR) plot
Model:

```
gr14 by bc0054@1 bc0056 (a12) !marker item "g1" lambda bl0054 bl0069 bl0076 (cl1-cl3); !cross-loadings [bc0054@0]; !marker item "g1" tau [bc0056] (b12);

gr16 by dc0054* dc0056* (a21-a22) dl0054* dl0069* dl0076* (cl4-cl6); !cross-loadings [dc0054 dc0056] (b21-b22);

gr18 by fc0054* fc0056* (a31-a32) fl0054* fl0069* fl0076* (cl7-cl9); !cross-loadings [fc0054 fc0056] (b31-b32);

gr20 by hc0054* hc0056* (a41-a42) hl0054* hl0069* hl0076* (cl10-cl12); !cross-loadings [hc0054 hc0056] (b41-b42);
```
he14 by b10054@1 b10069 b10076 (c12-c13)
bc0054 bc0056 (c13-c14);
[b10054@0];
[b10069 b10076] (d12-d13);

he16 by d10054* d10069* d10076* (c21-c23)
dc0054* dc0056* (c15-c16);
[d10054 d10069 d10076] (d21-d23);

he18 by f10054* f10069* f10076* (c31-c33)
fc0054* fc0056* (c17-c18);
[f10054 f10069 f10076] (d31-d33);

he20 by h10054* h10069* h10076* (c41-c43)
hc0054* hc0056* (c19-c20);
[h10054 h10069 h10076] (d41-d43);

i_gr  s_gr  |  gr14@0  gr16@1  gr18@2  gr20@3;
[i_gr  s_gr];
i_he  s_he  |  he14@0  he16@1  he18@2  he20@3;
[i_he  s_he];
Appendix

Mplus Input: approximate MI

Model priors:
   Do (2,2) diff (a1#-a4#) ~ N(0,0.01); !"do diff" for lambda -differences
   Do (2,2) diff (b1#-b4#) ~ N(0,0.01); !"do diff" for tau-differences

   a21 ~ N(1,0.01); ! priors for marker item "g1" lambda
   a31 ~ N(1,0.01);
   a41 ~ N(1,0.01);

   b21 ~ N(0,0.01); ! priors for marker item "g1" tau
   b31 ~ N(0,0.01);
   b41 ~ N(0,0.01);

   Do (2,3) diff (c1#-c4#) ~ N(0,0.01);
   Do (2,3) diff (d1#-d4#) ~ N(0,0.01);

   c21 ~ N(1,0.01);
   c31 ~ N(1,0.01);
   c41 ~ N(1,0.01);

   d21 ~ N(0,0.01);
   d31 ~ N(0,0.01);
   d41 ~ N(0,0.01);
Appendix

**Mplus Input: approximate MI**

Model constraint:

```
NEW(a11 ave1 diff1_1-diff1_4);  ! calculation of differences between
a11=1;
ave1=(a11+a21+a31+a41)/4;
Do(1,4) diff1_#=a#1-ave1;

NEW(b11 ave2 diff2_1-diff2_4);  ! calculation of differences between
b11=0;
ave2=(b11+b21+b31+b41)/4;
Do(1,4) diff2_#=b#1-ave2;

NEW(c11 ave3 diff3_1-diff3_4);  ! calculation of differences between
c11=1;
ave3=(c11+c21+c31+c41)/4;
Do(1,4) diff3_#=c#1-ave3;

NEW(d11 ave4 diff4_1-diff4_4);  ! calculation of differences between
d11=0;
ave4=(d11+d21+d31+d41)/4;
Do(1,4) diff4_#=d#1-ave4;
```